

Dear Workshop Participant,

Congratulations! You have just won the consolation prize in the Ohio State Lottery's mega jackpot, an invitation to attend Ted's workshop on cooperative learning, as applied to developmental math; a tool for increasing student success and retention.

I would like to thank you in advance for inviting me to your campus to spend the day sharing my enthusiasm and experiences with cooperative learning, especially in developmental math courses. I look forward to sharing our ideas, both yours and mine, on how to help our students succeed in math. Since outcome objectives are that latest focus in higher education I have two for the day: first is to bring home at least one or two activities, ideas or approaches that I can use in my classes, based upon your input; second, to encourage (convince) you to try a few of the coop structures we will work on today, in your classes, as soon as you can see a way to fit them into your syllabus.

Attached is an agenda for the day's activities. It may appear to be quite long. I hope to be able to have us work through all the activities listed since the math will not be time consuming. One of the questions that arises about coop learning is what do I do with students who finish activities early and my answer is to be over prepared with extra worksheets that provide more problems for the day's topic. That was my strategy in preparing this agenda. We will be able to modify the agenda during the day if we find a consensus to do so. The last topic is very interesting, so I hope we will get to the question of what your students are like? I have used this question as a basis for workshops at AMATYC and elsewhere with very positive results.

I have attached two additional items. A questionnaire that will form the basis for our first activity and help me get to learn about your innermost thoughts before we start the math related activities. Please complete it before the workshop. Second is an article comparing collaborative learning to cooperative learning that will provide a context for the day's discussions.

If you would like to contact me prior to the workshop please feel free to email me at [tpanitz@capecod.net](mailto:tpanitz@capecod.net) or call me at 508-428-4787. I would love to chat with you before the workshop to get a better idea of how I can tailor the day to meet your interests. I look forward to meeting you and working with you on this exciting area.

Regards,

Ted

## Cuyahoga Workshop –Participant Questionnaire

Why did you chose teaching as a career? What is your philosophy of teaching/learning? How do you teach? How do you involve your students in the learning process?

General biographical information- What information about you would help us get to know you better? How long have you been teaching? Where have you taught? Other pertinent experiences?

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Do you use cooperative learning activities or structures in your classes? How?

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What question(s) do you have about cooperative learning?

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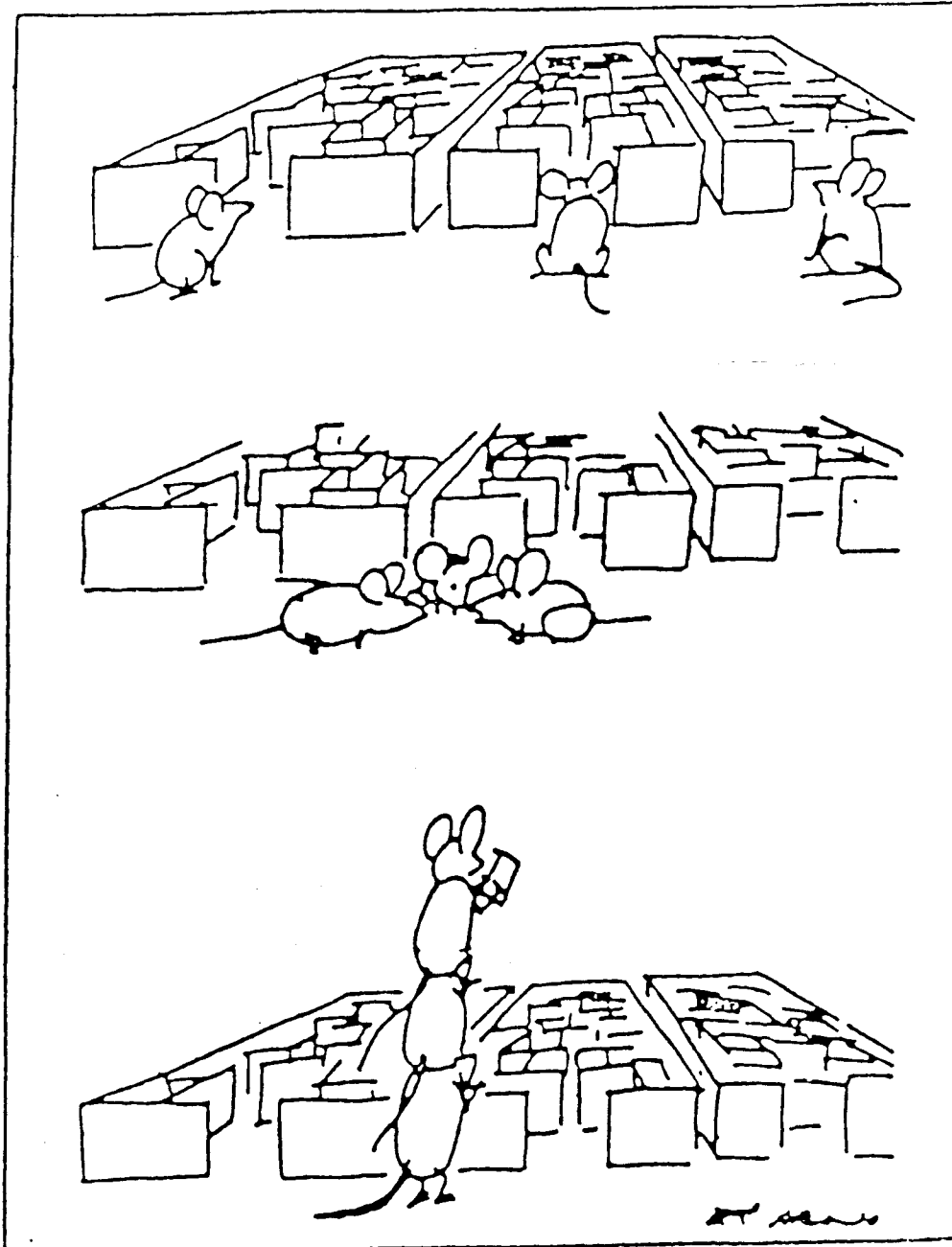
Please use the reverse side or extra sheets if you need more room to answer these questions.

**Cuyahogo Community College Workshop**  
**Exploring Cooperative Learning Techniques in Developmental Math Courses**  
**A Retention- Success Strategy**

The following structures and activities will be used to form a basis for our exploration, discussions, and Q&A about cooperative learning throughout the day

1. Ice Breaker- Introductions in Pairs (random organizing)- complete questionnaire before session starts
  - a. Sit with people whom you do not know real well, in groups of 4 please
  - b. Interview your new math buddy based on the questionnaire
  - c. Cognitive rehearsal- Introduce your new math buddy to the table
  - d. Whole class introductions-looking for common themes
2. Exploring orders of operations in pairs
  - a. Introduction to PEMDAS
  - b. Pairs worksheet with multiple problems- presentations of solutions to whole class
3. Pair Reading Explaining Structure- to explore class syllabus/textbook section
  - a. We will use this exercise to focus on mastery of results to table and then to the whole class
  - b. Reorganize groups by random process (count off by fours and regroup)
  - c. ice breaker- find 5 things everyone has in common not related to work or school
4. Jig Saw Structure- explore simple FOIL factoring in groups of 4.
  - a. Start with base group, reform into expert groups, expert groups teach their base group about their factoring case.
  - b. Pairs jig saw- complex foil factoring- trial and error versus grouping
5. Writing in Pairs- Write a procedure for the "grouping method" for factoring quadratics
6. Reorganize groups using birthdays and count off by fours (there will be a twist to this exercise). Ice breaker- find something that represents who you are. (from your wallet, pocketbook, pockets)
7. Polynomial/factoring review- individually /together
8. Pair writing- two column format- solving rational equations and linear equations
9. Time permitting- Pair/Quad/Whole group Discussion- What are your developmental student like?
10. Workshop feedback








# Co operative Learning



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# Students Retention

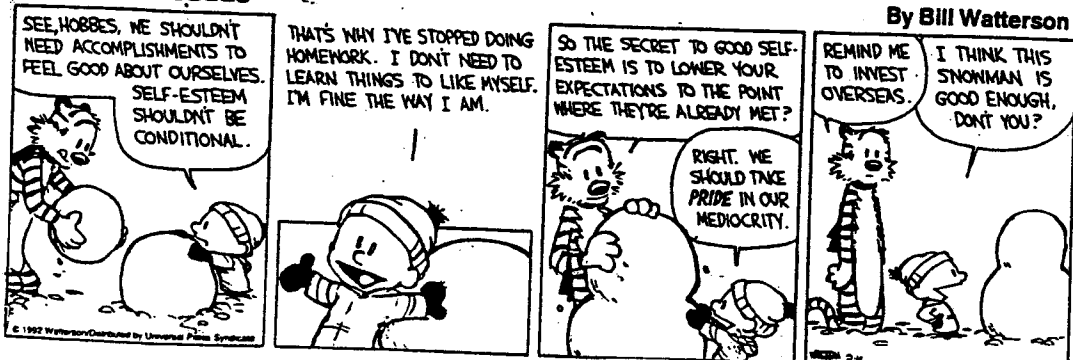
Studies have shown that students will retain:

-  10% of what they **READ**
-  20% of what they **HEAR**
-  30% of what the **SEE**
-  50% of what they **SEE & HEAR**
-  70% of what they **SAY**
-  90% of what they **SAY** as they **DO SOMETHING**
-  95% of what they **TEACH** to **SOMEONE ELSE**

CALVIN AND HOBBS



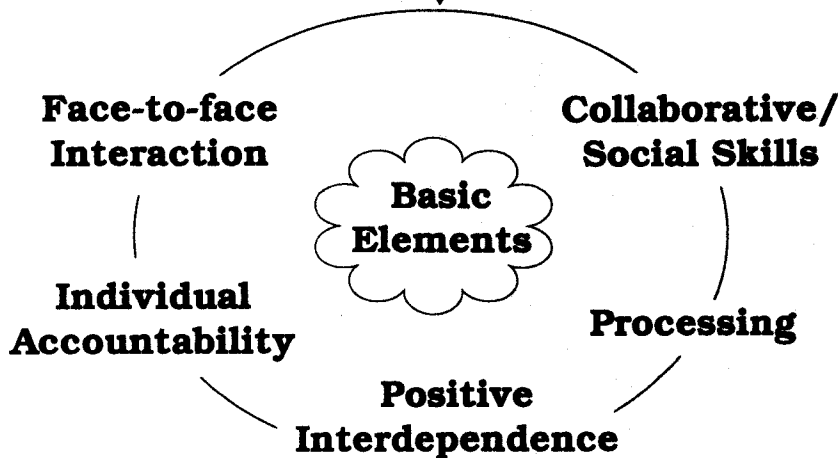
CALVIN AND HOBBS



# Cooperative Learning: An Overall Perspective



**Cooperative**



1. Goal
2. Incentive
3. Resource
4. Role
5. Sequence
6. Simulation
7. Outside Force
8. Environmental
9. Identity

<b>Role of the Teacher</b>	<b>Lesson Plan</b>	
	<b>Objectives</b>	<ul style="list-style-type: none"> <li>•Academic</li> <li>•Social Skills</li> </ul>
<b>I</b>	<b>Making Decisions Before Teaching</b>	
<b>II</b>	<b>Setting the Lesson</b>	
<b>III</b>	<b>Monitoring &amp; Intervening</b>	
<b>IV</b>	<b>Evaluating Product &amp; Process</b>	

## EXPLORING THE NEED FOR AN ORDER OF OPERATIONS

Determine two different ways to simplify each of the problems below.

1)  $5 + 3 \cdot 2$

possibility 1:

possibility 2:

2)  $16 \div 8 \cdot 2$

possibility 1:

possibility 2:

**What does the acronym PEMDAS stand for and how do we use it? Write your answer below.**

**There is a problem with using this acronym strictly. Can you think of what that problem is?**

SIMPLIFY EACH EXPRESSION USING THE CORRECT ORDER OF OPERATIONS- PEMDAS

1)  $3 \cdot 2 - 7 + 5$

2)  $18 - 4 \cdot 2 + 3 \cdot 3$

3)  $21 + 5 \cdot 4$

4)  $6(3 - 1) - 2(5 - 2)$

5)  $\frac{7+5}{6} + \frac{8+10}{9}$

6)  $5(2) + 7(4) - 2(7)$



7)  $16 \div 8 \cdot 4 + 36 \div 4 \cdot 2$

8)  $15 - 3 \cdot 4 + 7$

9)  $\frac{3(17-9)}{4} + \frac{9(16-7)}{3}$

10)  $\frac{7 \cdot 8 + 4}{5 \cdot 8 - 10} + \frac{9 \cdot 6 - 4}{6 \cdot 5 - 20}$

11)  $92 - 3[2(5-2)]$

12)  $16 \div 2 \div 4 \div 2$

## MAT030-ELEMENTARY ALGEBRA COURSE DESCRIPTION

**INSTRUCTOR-** Ted Panitz

**OFFICE-** 213 Science building

**PHONE-** 362-2131 ext 4421

**HOME** 428-4787

**E-MAIL** tpanitz@capecod.edu

**WEB PAGE:** <http://tpanitz.jimdo.com>

**WHAT TO EXPECT-** The course is designed to use a mastery approach to study math with elementary algebra. Students qualify to enter this course by one of two ways. Either they have completed Basic Math MAT020 at 4C's or they have scored high enough on the computer placement test to qualify for MAT030. In either case a basic arithmetic background is presumed. This means that students should be familiar with the BASIC MATH materials covered in chapter R and parts of chapter 1 of the textbook. We will do a short review of this material to help bring everyone up to the same level prior to beginning MAT030, however it is recommended that each student review this material on their own.

The very mention of the word algebra often evokes anxiety in people. This course has been developed with those people in mind. While not exactly self paced, this course will attempt to accommodate the fact that not all students learn at the same rate. For those people who have completed MAT020 it is hoped that your anxiety level has been reduced and your confidence in learning algebra increased proportionally. One purpose of this course is to help you become independent math learners by encouraging you to do math in class with your peers through collaborative learning activities. After you complete this class you will be qualified and competent to take intermediate algebra or any course with an elementary algebra prerequisite.

**METHOD OF GRADING-** Students may receive a letter grade or a pass/fail grade at their discretion. There are five masteries and a final exam. In order to obtain a P grade you need a minimum of **80% on all masteries** and 70% on the final. The final is cumulative. After you complete a test I will check it for you and indicate any incorrect answers and then allow you to correct any mistakes immediately. If you get above an 80% on the corrected test I will continue to return the test until you complete all the problems correctly. If you do not reach an 80% mastery level after you have made corrections you will need to restudy the material, complete the chapter review section showing all your work and submit it to me before you take another mastery, following the same procedure as above. Makeup masteries will be done outside of class time. Past experience shows that students who work all the text problems for homework and participate in class and attend every class rarely need to do a mastery retake.

At the end of the semester you may be asked to retake the CPT assessment as a post test. The purpose of this is to assist the math department in building a data base with pre and post test information from students who complete our developmental courses. In the event we make changes to the courses we need this information to determine if the changes were effective. In order to encourage you to participate I offer the following final exam option. If you take the CPT and receive a raw score of 58, and you have passed all the chapter masteries, then you will exempt the in class final exam. The take home final is still required in any case. If you do not complete the course requirements but get a score above 58 you meet the equivalent prerequisite for follow up courses.

**CLASS PROCEDURES-** Class attendance is very important. In addition to the class being fun we will cover important concepts in a way that will make algebra interesting and understandable. In addition you will meet nice people and make lifelong friends. Class participation is encouraged. Attendance is taken in each class. We work problems in small groups and in pairs, on the board and on work sheets together. My main objective in this course is to encourage you to learn how to learn algebra. I do not insist however that everyone participate publicly, such as going to the board. I want you to be comfortable with the process, so you may decline an invitation to participate any time you feel uncomfortable in doing so.

**This class does not meet graduation requirements** for math nor does it count toward graduation credits. It does count for institutional credits needed for financial aid, health insurance or other institutional requirements.

## ORDERS OF OPERATIONS- SIMPLIFYING PARENTHESES

**Simplify the following expression by having each person in your group (pair) do one step and then pass the paper to the next person. Do one operation, have everyone in the group check it to assure it is correct and then pass the paper to the next person in the group.**

1)  $10(6 - 5(8 - 4))$

2)  $(9(7 - 3) + 13) - (11 - 2(6 + 9))$

3)  $(6(y + 4) - 12) - (5(y - 8) + 11)$

4)  $3((8(3X - 2) + 9) + (7(X + 4) + 6X))$

5)  $3((6(a - 4) + 5) - 2(5(a + 8) - 10))$

6)  $5((3^2(4z - 3) + 2) - 4(-3(2z - 5) - 7))$

## Factoring Jigsaw - Group Exercise

There are four parts to the factoring jigsaw. Each part represents a different case for factoring simple quadratic expressions ( $ax^2 + bx + c$ ) A simple quadratic expression has the coefficient of the first term (a) equal to 1.

This exercise will require groups of 4 people who will serve as a base group. Each person in the base group will be given a different case of simple factoring to work on and become an expert on that case. You will not be alone in your quest to become an expert in your case. You will work with other people who have the same case, so you will need to form new temporary groups according to your case number. Then follow the directions below.

**Case #1-** The coefficients of the second and third terms in the quadratic expression (b and c) are positive.

Procedure:

1. Factor each quadratic expression. If you need help confer with your group members or the textbook.
2. Determine the pattern of signs and numbers in the original expression and also in the factored form representative of this case.
3. Develop a strategy, with your fellow experts, to help you to explain how to factor your expressions to your base group.
4. Make up 5 problems of your own that you will ask your base group members to solve after you have explained what is unique about your case.

1)  $X^2 + 10X + 21$  \_\_\_\_\_

2)  $Y^2 + 14Y + 45$  \_\_\_\_\_

3)  $A^2 + 15A + 44$  \_\_\_\_\_

4)  $T^2 + 12T + 36$  \_\_\_\_\_

5)  $X^2 + 24X + 144$  \_\_\_\_\_

**Make up 5 problems to take back for your base group to solve after you have provided them with your expert teaching presentation**

Original quadratic	factored quadratic
1) _____	_____
2) _____	_____
3) _____	_____
4) _____	_____
5) _____	_____

## Factoring Jigsaw - Group Exercise

There are four parts to the factoring jigsaw. Each part represents a different case for factoring simple quadratic Expressions ( $ax^2 + bx + c$ ). A simple quadratic expression has the coefficient of the first term (a) equal to 1. This exercise will require groups of 4 people who will serve as a base group. Each person in the base group will be given a different case of simple factoring to work on and become an expert on that case. You will not be alone in your quest to become an expert in your case. You will work with other people who have the same case, so you will need to form new temporary groups according to your case number. Then follow the directions below.

**Case #2-** The coefficient of the second term is positive and the third term is negative in the quadratic expression (b is positive and c is negative)

Procedure:

1. Factor each quadratic expression. If you need help confer with your group members or the textbook.
2. Determine the pattern of signs and numbers in the original expression and also in the factored form representative of this case.
3. Develop a strategy, with your fellow experts, to help you to explain how to factor your expressions to your base group.
4. Make up 5 problems of your own that you will ask your base group members to solve after you have explained what is unique about your case.

1)  $X^2 + 5X - 14$  \_\_\_\_\_

2)  $Y^2 + 4Y - 32$  \_\_\_\_\_

3)  $A^2 + A - 42$  \_\_\_\_\_

4)  $T^2 + 5T - 36$  \_\_\_\_\_

5)  $X^2 + 9X - 52$  \_\_\_\_\_

**Make up 5 problems to take back for your base group to solve after you have provided them with your expert teaching presentation**

Original quadratic	factored quadratic
1) _____	_____
2) _____	_____
3) _____	_____
4) _____	_____
5) _____	_____

## Factoring Jigsaw - Group Exercise

There are four parts to the factoring jigsaw. Each part represents a different case for factoring simple quadratic Expressions  $(ax^2 + bx + c)$ . A simple quadratic expression has the coefficient of the first term (a) equal to 1. This exercise will require groups of 4 people who will serve as a base group. Each person in the base group will be given a different case of simple factoring to work on and become an expert on that case. You will not be alone in your quest to become an expert in your case. You will work with other people who have the same case, so you will need to form new temporary groups according to your case number. Then follow the directions below.

**Case #3-** The coefficients of the second and third terms in the quadratic expression (b and c) are negative.

Procedure:

1. Factor each quadratic expression. If you need help confer with your group members or the textbook.
2. Determine the pattern of signs and numbers in the original expression and also in the factored form representative of this case.
3. Develop a strategy, with your fellow experts, that will help you to explain how to factor your expressions to your base group.
4. Make up 5 problems of your own that you will ask your base group members to solve after you have explained what is unique about your case.

1)  $X^2 - 4X - 45$  \_\_\_\_\_

2)  $Y^2 - 5Y - 14$  \_\_\_\_\_

3)  $A^2 - 7A - 44$  \_\_\_\_\_

4)  $T^2 - 5T - 36$  \_\_\_\_\_

5)  $X^2 - 4X - 21$  \_\_\_\_\_

**Make up 5 problems to take back for your base group to solve after you have provided them with your expert teaching presentation**

Original quadratic	factored quadratic
1) _____	_____
2) _____	_____
3) _____	_____
4) _____	_____
5) _____	_____

## Factoring Jigsaw - Group Exercise

There are four parts to the factoring jigsaw. Each part represents a different case for factoring simple quadratic expressions ( $ax^2 + bx + c$ ). A simple quadratic expression has the coefficient of the first term (a) equal to 1. This exercise will require groups of 4 people who will serve as a base group. Each person in the base group will be given a different case of simple factoring to work on and become an expert on that case. You will not be alone in your quest to become an expert in your case. You will work with other people who have the same case, so you will need to form new temporary groups according to your case number. Then follow the directions below.

**Case #4-** The coefficient of the second term is negative and the third term is positive in the quadratic expression (b is negative and c is positive)

Procedure:

1. Factor each quadratic expression. If you need help confer with your group members or the textbook.
2. Determine the pattern of signs and numbers in the original expression and also in the factored form representative of this case.
3. Develop a strategy, with your fellow experts, that will help you to explain how to factor your expressions to your base group.
4. Make up 5 problems of your own that you will ask your base group members to solve after you have explained what is unique about your case.

1)  $X^2 + 10X + 21$  \_\_\_\_\_

2)  $Y^2 + 14Y + 45$  \_\_\_\_\_

5)  $A^2 + 15A + 44$  \_\_\_\_\_

6)  $T^2 + 12T + 36$  \_\_\_\_\_

5)  $X^2 + 24X + 144$  \_\_\_\_\_

**Make up 5 problems to take back for your base group to solve after you have provided them with your expert teaching presentation**

Original quadratic	factored quadratic
1) _____	_____
2) _____	_____
3) _____	_____
4) _____	_____
5) _____	_____



## Factoring Jigsaw - complex quadratic expressions $(ax^2 + bx + c)$ (a is not equal to 1)

In this exercise you will work in pairs to become experts on either 1) the grouping method or 2) the trial and error method for factoring complex quadratic expressions

### Factoring complex quadratic expressions using the trial and error or foil method

Factor each expression below using the trial and error method in order to help yourself become an expert in this procedure. Work with your partner to develop an explanation that you will use to explain this procedure to the other members of your group. They are working on the “grouping method” and will explain that method to you after they become experts in that technique.

1)  $2Y^2 + 7Y + 3$

2)  $5Y^2 - 22Y + 8$

3)  $6B^2 - 17B + 15$

Work with your partner to develop a strategy for explaining the trial and error (foil) approach to factoring. Then teach your other partners this factoring technique.

## Factoring Jigsaw - complex quadratic expressions ( $ax^2 + bx + c$ ) (a is not equal to 1)

In this exercise you will work in pairs to become experts on either 1) the grouping method or 2) the trial and error method for factoring complex quadratic expressions

### Factoring complex quadratic expressions using the “grouping” method

Factor each expression below using the grouping method in order to help yourself become an expert in this procedure. Work with your partner to develop an explanation that you will use to explain this procedure to the other members of your group. They are working on the “trial and error method” and will explain that method to you after they become experts in that technique.

$$4) 2Y^2 + 7Y + 3$$

$$5) 5Y^2 - 22Y + 8$$

$$6) 6B^2 - 17B + 15$$

Work with your partner to develop a strategy for explaining the grouping approach to factoring. Then teach your other partners this factoring technique.

# DEVELOPING A PROCEDURE FOR FACTORING USING THE GROUPING METHOD

(If you use the trial and error method you may not pass go and you may not collect \$200)

Work out the problem in the left column and write out each step in the left column as you proceed

Polynomial to be factored

Factoring by grouping procedure

1) $6W^2 + 19W + 10$	1) _____ 2) _____ _____ 3) _____ _____ 4) _____ _____
2) $10X^2 + 11X = 6$	1) _____ 2) _____ _____ 3) _____ _____ 4) _____ _____
3) $8m^2 - 10m - 3$	
4) $3x^2 + 16x + 16$	

**Worksheet 4- chapter 6**  
**Comparing multiplying and factoring methods**

**MULTIPLY OR FACTOR EACH COLUMN. CAN YOU SEE ANY PATTERNS BETWEEN THE TWO OPERATIONS?**

Multiply this column	factor this column
$4(3A - 5B + 6C)$	$12A - 20B + 24C$
$7(2X^2 - 5X - 4)$	$X^2 - 35X - 28$
$4Y^3(5Y - 6)$	$20Y^4 - 24Y^3$
$3X^4(4X^2 - 7X - 8)$	$12X^6 - 21X^5 - 24X^4$
$-4T(7T^3 + 4T^2 - 4T + 1)$	$-28T^4 - 16T^3 + 16T^2 - 4T$
$3X^2Y^5(2XY^3 - 4X^3Y^2 - 1)$	$6X^3Y^8 - 12X^5Y^8 - 3X^2Y^5$
$7(3A + 5B - 4C)$	$21A + 35B - 28C$
$3(3X^3 - 4X^2 - 10)$	$9X^3 - 12X^2 - 30$
$2T^2(T^3 - 4)$	$2T^5 + 8T^2$
$5Y^2(Y^5 - 4Y^2 - 3)$	$5Y^7 - 20Y^4 - 15Y^2$
$4X(2X^3 - X^2 + 3X - 1)$	$8X^4 - 4X^3 + 12X^2 - 4X$
$6X^2Y^2(Y^2 - 6X^2 + 1)$	$6X^2Y^4 - 18X^4Y^2 + 6X^2Y^2$
$4B^3(6B^3 - 8B^2 - 3B + 2)$	$24B^6 - 32B^5 - 12B^4 + 8B^3$
$-5S^3T^3(4S - 3ST + 4T)$	$-20S^4T^3 - 15S^4T^4 - 20S^3T^4$
$7X(2X^2 + X - 3)$	$14X^3 + 7X^2 - 21X$
$8XY(X^3Y^6 - 2X^2 - 3Y)$	$8X^4Y^7 - 16X^3Y - 24XY^2$
$7(3A + 5B - 7C + 8E + 4F)$	$21A + 35B - 49C + 56E + 28F$
$3Y^5(2Y^5 - 3Y^4 - 5Y^3 + 3Y^2 - 7Y - 6)$	$6Y^{10} - 9Y^9 - 15Y^8 + 9Y^7 - 21Y^6 - 18Y^5$
$8A(2A^6 - 3A^5 - 5A^4 + A^3 - 4A^2)$	$16A^7 - 24A^6 - 40A^5 + 8A^4 - 32A^3$
$9(2X^4 + 9X^3 - 3X^2 - 4X - 5)$	$18X^4 + 81X^3 - 27X^2 - 36X - 45$

Use the smallest exponent on any variable to factor the common variable.

You must factor completely!!!

If there is a variable left in every term after you think you are finished then you have not factored completely.

## Multiplying conjugates or factoring difference of squares

Complete the table. Use separate paper if you need to. What patterns do you see?

Multiply this column	factor this column
$(x + 3)(x - 3)$	$x^2 - 9$
$(y + 2)(y - 2)$	$y^2 - 4$
$(a - 6)(a + 6)$	$a^2 - 36$
$(z - 10)(z + 10)$	$z^2 - 100$
$(2x - 5)(2x + 5)$	$4x^2 - 25$
$(3T + 8)(3T - 8)$	$9T^2 - 64$
$(T + 4)(T - 4)$	$T^2 - 16$
$(x + 2)(x - 2)$	$x^2 - 4$
$(p - 5)(p + 5)$	$p^2 - 25$
$(y - 9)(y + 9)$	$y^2 - 81$
$(5c + 1)(5c - 1)$	$25c^2 - 1$
$(6d - 5)(6d + 5)$	$36d^2 - 25$
$(4m + n)(4m - n)$	$16m^2 - n^2$
$(7w - 10y)(7w + 10y)$	$49w^2 - 100y^2$
$(10x - 9y)(10x + 9y)$	$100x^2 - 81y^2$
$(4m + 7n)(4m - 7n)$	$4m^2 - 49n^2$
$(4y - 3)(4y + 3)$	$16y^2 - 9$
$(5a + 6)(5a - 6)$	$25a^2 - 36$
$(5a + 4b)(5a - 4b)$	$25a^2 - 16b^2$
$(6t + 3)(6t - 3)$	$36T^2 - 9$

Multiplying- Subtract the square of the second term in the parenthesis from the square of the first term

Factoring- Use the square root of each term in the expression and add and subtract each in separate parenthesis

## SQUARING AND FACTORING BINOMIALS

Multiply this column	factor this column
$(X + 4)^2$	$x^2 + 8x + 16$
$(Y + 1)^2$	$y^2 + 2y + 1$
$(2P - 3)^2$	$4p^2 - 12p + 9$
$(5T - 6)^2$	$25T^2 - 60T + 36$
$(7W + 4)^2$	$49W^2 + 56W + 16$
$(M - 6)^2$	$M^2 - 12M + 36$
$(X + 5)^2$	$X^2 + 10X + 25$
$(y - 3)^2$	$y^2 - 6y + 9$
$(z - 7)^2$	$z^2 - 14z + 49$
$(p - 1)^2$	$p^2 - 2p + 1$
$(4x + 5)^2$	$4x^2 + 20x + 25$
$(5w - 9)^2$	$25w^2 - 90w + 81$
$(10T + 7w^2)^2$	$100T^2 + 140TW + 49W^2$
$(3x - 4y)^2$	$9x^2 - 24xy + 16y^2$
$(x + 6y)^2$	$x^2 + 12xy + 36y^2$
$(3a + 5b)^2$	$9a^2 + 30ab + 25b^2$

To multiply mentally square the first term to get the first term in the answer, double the product of the first and second term in the parenthesis to get the middle term in the answer, square the second term in the parenthesis to get the end term in the answer.

To factor when the coefficient is = 1 take half the middle term and place it in the parenthesis which is then squared. For this to work half the middle term squared must equal the end term.

For a not equal to 1, take the square root of the end terms and if they add up to the middle term when multiplied and doubled these are your factors to be placed in a parenthesis which is squared.

## MULTIPLYING BINOMIALS AND FACTORING TRINOMIALS

Complete the table. Use separate paper if you need to. What patterns do you see?

Multiply this column	factor this column
$(x + 4)(x + 5)$	$x^2 + 9x + 20$
$(Y + 1)(Y + 3)$	$Y^2 + 4Y + 3$
$(T + 5)(T - 2)$	$T^2 + 3T - 10$
$(X - 4)(X - 5)$	$X^2 - 9X + 20$
$(Y - 8)(Y + 3)$	$Y^2 - 5Y - 24$
$(X - 7)(X - 8)$	$X^2 - 15X + 56$
$(x + 4)(x + 3)$	$x^2 + 7x + 12$
$(y + 5)(y + 3)$	$y^2 + 8y + 15$
$(z - 8)(z + 2)$	$z^2 - 6z - 16$
$(p - 7)(p + 5)$	$p^2 - 2p - 35$
$(x - 7)(x - 3)$	$x^2 - 10x + 21$
$(p + 5)(p - 2)$	$p^2 + 3p - 10$
$(p + 2q)(p + 5q)$	$p^2 + 7pq + 10q^2$
$(X - 3Y)(X - 5Y)$	$X^2 - 8XY + 15Y^2$
$(x + 5y)(x + 2y)$	$x^2 + 7xy + 10y^2$
$(T - 4S)(T + 3S)$	$T^2 + TS - 12S^2$

**Multiplying-** Add the two numbers in the parenthesis to get the middle term of the answer and multiply the two numbers to get the end term of the foil multiplication.

**Factoring-** Find two numbers that are factors of the end term that add up to the middle term and multiply to get the end term of the original problem

## COMPLEX TRINOMIALS

Multiply this column	factor this column
$(4X - 1)(X + 2)$	$4X^2 + 7X - 2$
$(3A + 2)(2A - 5)$	$6A^2 - 11A - 2$
$(2P - 3)(p + 1)$	$2P^2 - P - 3$
$(5T - 6)(2T - 7)$	$10T^2 - 47T + 42$
$(7W + 4)(W - 8)$	$7W^2 - 52W - 32$
$(M - 6)(4M - 3)$	$4M^2 - 27M + 18$
$(3X - 4)(X + 1)$	$3X^2 - X - 4$
$(2Y - 3)(3Y + 1)$	$6Y^2 - 7Y - 3$
$(5T - 3)(2T + 3)$	$10T^2 + 9T - 9$
$(4X - 3)(3X - 5)$	$12x^2 - 29x + 15$
$(2Y + 5)(Y + 1)$	$2y^2 + 7y + 5$
$(3Z + 4)(Z + 2)$	$3z^2 + 10z + 8$
$(10t + w)(2t - 3w)$	$20T^2 - 28TW - 3W^2$
$(3x - y)(5x + 4y)$	$15X^2 - 7XY - 4Y^2$
$(a + 5b)(a - b)$	$a^2 + 4ab - 5b^2$
$(5x + 1)(x - 2)$	$5x^2 - 9xy - 2y^2$

Trial and error method: pick numbers that create the first term of the quadratic and the last term and then check to see if the middle term works out correctly when you FOIL the answer



## PAIR EXERCISE- TWO COLUMN FORMAT COMBINING ENGLISH AND MATH

With your partner, discuss each step of the process of solving the given equation and explain your steps in English in the right hand column. Try to reach an agreement on the language you wish to use BEFORE writing your explanations.

$\frac{X}{X-4} - \frac{2}{X+3} = \frac{20}{X^2 - X - 12}$	<p>This is the starting equation. Explain what procedures are needed to get to the next step</p>
$\frac{X}{X-4} - \frac{2}{X+3} = \frac{20}{(X-4)(X+3)}$	
$\text{LCD} = (X+3)(X-4)$	
$(X+3)(X-4) \left( \frac{X}{(X-4)} - \frac{2}{(X+3)} \right) = \frac{20(X+3)(X-4)}{(X-4)(X+3)}$	
$X^2 + 3X - 2X + 8 = 20$	
$X^2 + X - 12 = 0$	
$(X+4)(x-3) = 0$	
$X = -4$ $X = 3$	





# SOLVING EQUATION USING ALGEBRA AND ENGLISH

The following equation is worked out by completing each operation as a separate activity. Explain in english on the right side of the page what was done on each line to work towards solving the equation.

ALGEBRA SOLUTION	ENGLISH EXPLANATION
$5X + 3(2X - 4) = 7(-5X + 1)$	This is the starting equation.
$5X + 6X - 12 = 7(-5X + 1)$	
$11X - 12 = -35X + 7$	
$46X - 12 = 7$	
$46X = 19$	
$X = 19/46$	



# JIG SAW EXERCISE

**There are two different and distinct methods for creating an equation of a line when you are given two points from that line.**

**In this exercise you will find another person in the room who has the same assignment or jigsaw part number.**

**You will then work together to become experts in your method. To do this you will complete the given problems and then outline in writing your process or strategy.**

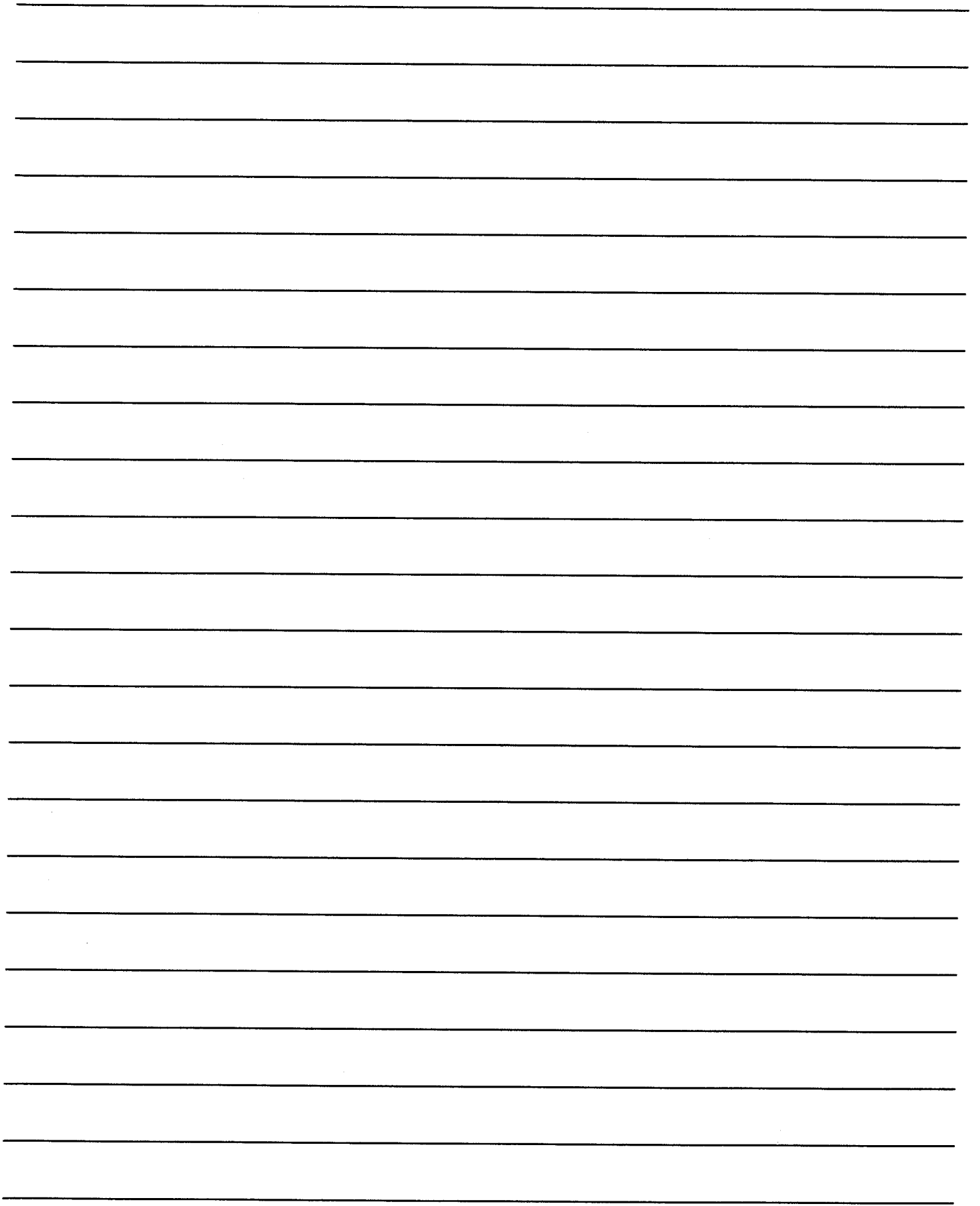
**Next you will return to your original partner and teach that person your method.**

**Jigsaw part 1-- using the formula  $y = mx + b$  create an equation of a line which contains the following points:**

**1) (5, 2) and (3, 1)**

**2) (-4, -2) and (6, 3)**

**Use the reverse side to write out your explanation. This will help your partner understand your method better. Yes it will!!!!**



# JIG SAW EXERCISE

There are two different and distinct methods for creating an equation of a line when you are given two points from that line.

In this exercise you will find another person in the room who has the same assignment or jigsaw part number.

You will then work together to become experts in your method. To do this you will complete the given problems and then outline in writing your process or strategy.

Next you will return to your original partner and teach that person your method.

Jigsaw part 2-- using the formula  $y - y_1 = m(x - x_1)$  create an equation of a line which contains the following points:

1) (5, 2) and (3, 1)

3) (-4, -2) and (6, 3)

Use the reverse side to write out your explanation. This will help your partner understand your method better. Yes it will!!!!



