**Using Cooperative Learning 100% Of the Time In Mathematics Classes Establishes A Student-centered, Interactive Learning Environment**

By Dr. Theodore Panitz

Cape Cod Community College

**Introduction and rationale for using cooperative learning**

College teachers continue to deal with mathematics curriculum reform in order to make mathematics more relevant and meaningful for students, encourage students to think critically, and make the process of learning mathematics more student centered and interactive. AMATYC has established a set of standards for mathematics education before Calculus. The AMATYC Standards (Cohen1995) include a number of references for the need for learner centered approaches, such as cooperative learning paradigms, in mathematics education. The Standards, however, do not prescribe specific methods for implementation of cooperative learning paradigms. This article attempts to fill the gap by describing one approach to implementing cooperative learning in mathematics classes, virtually 100% of the time. By describing this maximum approach it is hoped that teachers will be encouraged to try cooperative learning at least part of the time in their classes. The use of cooperative structures this extensively does not preclude the use of mini-lectures or professor lead whole class discussions. The use of a short presentation can be very effective when it is tailored to students' needs. Cooperative activities help identify widespread student misconceptions, and enable the teacher to focus on specific concepts or algorithims. The process outlined below was developed mainly in developmental math classes. It is also used by the author in college level mathematics and engineering classes at Cape Cod Community College.

The AMATYC Standards call for the use of student centered, interactive learning strategies for teaching mathematics. For example, under basic principles the Standards state: "Mathematics must be taught as a laboratory discipline. Effective mathematics instruction should involve active student participation. In-depth projects employing genuine data should be used to promote student learning through guided hands-on investigation." (p5) "These standards emphasize problem solving, the use of technology, intuitive understanding, and collaborative learning strategies." (p5) Standard 1-5, Communicating states: "Students will acquire the ability to read, listen to, and speak mathematics. Students will acquire the skills necessary to communicate mathematical ideas and procedures using appropriate vocabulary and notation. Students will learn to read and listen to mathematical presentations and arguments with understanding. Furthermore, mathematics faculty will adopt instructional strategies that develop both oral and written communication skills within a context of real applications relevant to the particular group of students." (p11) Standard P-2, Interactive and collaborative learning, states: "Mathematics faculty will foster interactive learning through student writing, reading, speaking and collaborative activities so that students can learn to work effectively in groups and communicate about mathematics both orally and in writing." (p16) The section on pedagogy states: "The use of cooperative learning strategies is also critical to providing positive learning experiences. Many students at this level have low self-esteem. Faculty must avoid reinforcing student perceptions that the teacher is the sole authority and that the student cannot learn except through the teacher. As faculty take on the role of coach, rather than authority figure, and as students learn to work together, they will begin to realize the mathematical power they posses." (p28) "If teachers are to make problem solving central to learning mathematics they must take risks. They need to feel confident in their knowledge of mathematics, be willing to explore new mathematical ideas, and be able to stimulate active discourse in the classroom. Faculty who teach mathematics courses for prospective teachers must nuture this spirit of active inquiry. In addition, since it is common for teachers to teach the way they were taught, faculty must use in their own classes the instructional techniques that prospective teachers will be expected to use." (p46) Institutional recommendations state: "Improvements in pedagogy are necessary for the student to become a more active, involved learner. Learning and problem solving through teamwork in the mathematics classroom must reflect the team approach to problem solving and communication in the world of work." (p63)

The Fall 1998 AMATYC Review presented an article on research which

demonstrated the effectiveness of cooperative learning on the achievement of beginning algebra students at four private liberal arts colleges in different regions throughout the country (Rupenow & Bogenshield (1998)). Johnson and Johnson (1989) reviewed over 600 studies on cooperative learning approaches which demonstrate the effectiveness of this approach to teaching and learning. Cooperative learning paradigms set very high expectations that students can understand course content by taking responsibility for their learning. In the process I describe below cooperative learning is used virtually 100% of the time in each class thus establishing that the students can indeed learn mathematics with minimal intervention from the instructor.

Methodology

To set the tone of the class I send my students a letter, two weeks prior to the beginning of the semester, in which I include a humorous introduction to the class and cooperative learning, a course syllabus, and a writing assignment in the form of a mathematical autobiography. I ask them to read the first chapter and start working on the text problems. The first chapter includes review materials from the prerequisite course. My intent is to emphasize the students' responsibility in the learning process well before the class starts and to demonstrate my own interest in helping them become independent mathematics learners. I include my home and work phone numbers to encourage students to contact me if they have any questions or concerns about working in groups or studying material before class.

On the first day of the semester I distribute a class schedule which specifies exactly which textbook sections they are responsible for on a given day. Students are encouraged to read the text before each class. I also ask the students to attempt to solve as many problems from the section exercises as possible, prior to class. They have student manuals which provide worked out solutions for all the even numbered problems plus answers to the odd numbered problems are in the text. Students are encouraged to check their homework solutions prior to class by going to the mathematics tutorial center or by consulting with classmates.

Students are seated at tables with four to six to a group. At the beginning of each class I distribute work sheets which contain problems or questions covering the day's topic or cooperative activity. Students first attempt to work the worksheet problems in pairs. They progress from simpler problems to more complex problems. Students working in pairs provides an optimal learning environment because one student is explaining and one is listening. Thus, all the students in the class are participating actively by listening or talking about mathematics. After the pairs complete the assignment they share their results with the other pairs of students at their table. Sharing results between pairs provides additional repetition and feedback for the students. I place the problems on the board in sequence and ask groups to present their solutions to the class. I also ask students to work directly out of the text together. We have a workbook form of text which encourages a hands on, interactive approach to studying mathematics. When students read the text together and explain sections to their partners they have an excellent opportunity to build their mathematics vocabulary.

During each class I circulate around the room observing each group's progress. I make suggestions on how they might go about finding the answers to their questions. I do not directly answer questions initially. Instead I encourage the students to use their text as a resource as well as any other student or group in the class. Those who did not do the reading and practice problems beforehand have an opportunity to do so at this point in the class. If enough students appear to be having difficulty or are making fundamental mistakes, I will ask for volunteers to put their solutions on the board and explain and defend their methodology. The advantage in this approach is that the explanation comes from the students peers. After the student explanations the groups go back to work and try to resolve whatever questions remain. If they are still confused, I facilitate a whole class discussion which usually elicits the source of their confusion.

In addition to the worksheets I often give group quizzes as a form of review. First each student works individually. Next they compare answers within their groups and try to reach agreement. At this point it becomes clear to me which students are competent and which are not. I can then encourage those who need it to get extra help outside of class.

Cooperative learning allows for a degree of flexibility in content coverage. On occasion I will postpone a test when I observe that a majority of students have not mastered the material. With all the outside pressures students face today, there occasionally develops a critical mass that just aren't ready to demonstrate their knowledge through a test, given at a time specified by the chronological sequence of a syllabus. My courses are not open ended. We continue covering the syllabus while the students review the material which they will be tested on. Students are encouraged to get extra help through the Mathematics Lab or other tutorial agencies on campus, by working with their peers in study groups, or by making arrangements to see me. By negotiating test schedules the students become more involved in establishing the course procedures and thus empowered to control their learning environment.

Finally, I give an in class test which is completed individually to maintain the accountability of each student. I use a mastery approach where students have an opportunity to correct their mistakes during the exam, before a final grade is calculated. While the students work on the exam I walk around the room observing their progress. When they have completed their test, I check it immediately and circle any incorrect answer, without indicating what mistake was made. The students then have an opportunity to make corrections. If they get below an 80% after corrections, then they are required to take retake a new test outside of class using the same procedure. If they get above an 80%, then I encourage them to continue making corrections until they have the test completely corrected. Their grade is based upon the final corrected test.

Every step of this cooperative learning paradigm is intended to encourage the students to take responsibility for their learning. This sets very high expectations for the students and myself as the course facilitator. I need to provide materials which will help guide the students through the process and I work with them to develop appropriate group interaction skills. I am intensely involved in each class as I circulate around the room, talk to students individually, in pairs, or in larger groups. I also guide the classes between whole group discussions and individual work. Students respond that the classes fly by, that they are exhausted at the end of class, but feel good about what they have accomplished. By the end of the semester the better students have learned how to become independent learners, their math phobia has all but disappeared and they actually begin to like math, and the less motivated students have learned more math than they ever expected. In class the students actively work through the content and obtain an understanding in a way that makes sense to them because they are developing their own solutions.

The procedures described above have evolved over a long period of time through a process of trial and error. I would not recommend that new teachers initiate an extensive cooperative learning system without first participating in training programs and conferences dealing with cooperative learning techniques. It takes time for teachers to develop a comfort level and develop a degree of confidence with cooperative processes. A good approach to incorporating cooperative learning in math classes would be to initiate one or two new techniques each semester until a full repertoire of activities is available to chose from.

**Examples of cooperative activities**

In order to give readers an idea of how cooperative learning is implemented, I will describe three activities: Pair-Reading and Math-Olympics may be used to cover any content area; Factoring-Jig-Saw was developed for a specific content area.

Pair-Reading- Pairs of students work together on this exercize. First, both students read the same section of the text or instructor provided materials. Next, one student explains a single paragraph or short section of the text to their partner. The partner listens, then asks questions if they do not understand the explanation. The listener then rephrases what they heard. The students alternate roles of "explainer" and "listener" until they complete all the material being studied. When the entire class has completed the exercize I ask groups at random to explain the material to the whole class. This serves as a check to make sure the students do indeed understand the material they are reading. In order to prepare the students for this activity I have the students Pair-Read the syllabus during the second class. The syllabus describes the cooperative nature of the class, the mastery approach to testing, grading, attendance policies, and other topics pertinent to the operation of the class. I have found that students initially read through course materials very casually, often missing key elements of course policies. This activity causes students to read through the syllabus carefully and to think critically about each element because they must explain each paragraph to their partner or listen to an explanation. Additional purposes of this activity include encouraging students to work with their neighbors and begin the process of training students in cooperative learning.

Math Olympics- This activity can be used with any content where multiple problems are involved. It is especially useful for chapter reviews or section practice. I use this activity to introduce solving equations in elementary algebra classes. The class is divided in groups of four. Existing groups may be used or new groups formed. I place five questions on the board. I use one more question then there are students in the group. Using one extra problem than there are group members avoids having the groups simply divide up the questions, one for each student. After 5-10 minutes, depending on the complexity of the problems, I ask each group to send one student to the board to record their group's answers, on a grid which I have drawn on the board. I check all the answers. The process can repeated for the duration of the class or a portion only. If I see that groups are having trouble with a set of problems I will stop the activity and facilitate a whole class discussion, or give a mini-lecture on the material. Students are actively involved in solving many problems in a short period of time during class. Groups are encouraged to work out their own processes for solving each set of problems. Thus, the students assume some of the responsibility for the class process. I have an opportunity to observe the students solving problems individually and in groups.

Factoring-Jig-Saw- This activity can be used where ever material can be segmented into separate components. Each group member becomes an expert on a different concept or procedure and teaches their concept to the group. I use this activity when covering factoring of polynomials, where the coefficient of the first term is one. There are four unique cases. The second and third coefficients may be both positive or both negative or have opposite signs. I form base groups of four students. Students count off from one to four. I distribute a worksheet for each case. The worksheets have five sample polynomials which I have made up for the students to factor, plus a space for each student to make up five problems of their own. Students reform groups by combining with other students who have been assigned the same case number, again four to a group. The students work together to determine what is unique about their case. They are in effect becoming experts in their case. The next step is for the students to develop a teaching strategy to bring back to their base groups. This is the stage where they make up their own problems. Each student practices his/her explanation with the case group. Finally, the students return to their base groups and teach their case. There is no preconceived way in which students must teach their material so the results are quite varied. This activity helps students understand what teaching mathematics involves as well as it provides an interesting and often entertaining class. Students learn how to work with different partners and begin to see that they can indeed assume responsibility for their own learning.

**Benefits of cooperative learning approaches**

Cooperative learning (CL) techniques, when used extensively, generate many advantages for the students and teachers. For an extensive review of the benefits of cooperative learning refer to Panitz & Panitz (1998) and Panitz (1999). For resders who wish to explore the nature and benefits of cooperative learning the following conclusions are presented along with references which contain detailed explanations and research results.

CL DEVELOPS HIGHER LEVEL THINKING SKILLS (Webb 1980, 1982).

CL STIMULATES CRITICAL THINKING AND HELPS STUDENTS CLARIFY IDEAS THROUGH DISCUSSION AND DEBATE (Johnson 1971, 1973, Johnson & Johnson 1987a, 1990)

SKILL BUILDING AND PRACTICE CAN BE ENHANCED AND MADE LESS TEDIOUS THROUGH CL ACTIVITIES IN AND OUT OF CLASS (Tannenberg 1995)

CL CREATES AN ENVIRONMENT OF ACTIVE, INVOLVED, EXPLORATORY LEARNING (Slavin 1990)

CL ENCOURAGES STUDENT RESPONSIBILITY FOR LEARNING (Baird & White 1984)

CL INVOLVES STUDENTS IN DEVELOPING CURRICULUM AND CLASS PROCEDURES (Kort 1992; Meier, M & Panitz, T., 1996)

CL PROMOTES HIGHER ACHIEVEMENT AND CLASS ATTENDANCE (Hagman & Hayes 1986)

Cl PROMOTES A POSITIVE ATTITUDE TOWARD THE SUBJECT MATTER(Bligh1972).

CL PROMOTES INNOVATION IN TEACHING AND CLASSROOM TECHNIQUES (Slavin 1980, 1990)

Cl FOSTERS MODELLING OF PROBLEM SOLVING TECHNIQUES BY STUDENTS' PEERS (Schunk & Hanson 1985)

WEAKER STUDENTS IMPROVE THEIR PERFORMNCE WHEN GROUPED WITH HIGHER ACHIEVING STUDENTS (Cohen 1986, 1994; Swing and Peterson 1982)

CL ADDRESSES LEARNING STYLE DIFFERENCES AMONG STUDENTS (Midkiff & Thomasson 1993)

CL IS ESPECIALLY BENEFICIAL IN MATHEMATICS COURSES. (Davidson 1990)

CL PROVIDES THE FOUNDATION FOR DEVELOPING LEARNING COMMUNITIES WITHIN INSTITUTIONS AND IN COURSES (TINTO 1997)

CL BUILDS SELF ESTEEM IN STUDENTS (Johnson & Johnson 1989)

CLASSROOM ANXIETY IS SIGNIFICANTLY REDUCED WITH CL (Kessler, Price & Wortman 1985)

TEST ANXIETY IS SIGNIFICANTLY REDUCED (Johnson & Johnson 1989)

CL PROVIDES A BASIS FOR ALTERNATE FORMS OF ASSESSMENT (Rosenshine & Stevens 1986) SUCH AS OBSERVATION OF GROUPS (PANITZ AND PANITZ (1996), GROUP SELF ASSESSMENT (JOHNSON & JOHNSON 1987), AND SHORT INDIVIDUAL WRITING ASSESSMENTS (Cross & Angelo 1993)

**Student responses to cooperative learning**

In order to give readers a feeling for student's reactions to cooperative learning, I have presented three student responses to a student self evaluation which I require at the end of each semester. I ask the students to evaluate their progress in the course, identify any changes they have made in their approach to learning mathematics or their attitude about mathematics. I specifically ask them to provide me with feedback on their reactions to my cooperative learning strategy. The following responses are representative of the many evaluations I have received over the years.

**Student #1**

"In the past as you know, Ted, I have taken a class with you and have enjoyed your approach in learning the material. Before your classes I disliked math. I was always getting aggravated and scared by it. Working together with those around me in a group has been a great help in understanding the material and the many different ways in which a problem can be tackled and solved. For me the beauty was being able to work one on one with someone every day. I was constantly learning something new and leaving class feeling relaxed and in control. On those days I could not understand something I did not feel half as bad as I normally would have, I knew that if it were something I could not figure out at home or at the next class period I could count on receiving help."

"If I had a chance to do it all over again I would do what I always say I am going to do at the beginning of the semester, and what I had wished I had done more of at the end of the semester, to do more math work at home by myself. With other classes, a job and still living at home it is hard. This semester was particularly hard for me. My grandfather got very sick and passed away a couple of weeks ago. For a couple of classes in a row I was absent and had difficulty keeping up with the material. I have visited the math lab recently and have received help there and am working on getting everything under way for the upcoming final."

**student #2**

"When I re-entered school almost two years ago I was told that I needed to take an algebra course. I paniced. Even though I had taken a large number of math courses in high school I feared that I had forgotten everything I had learned so long ago. It was a pleasure to realize how much I truly enjoy working with numbers once again. The course was presented in a way that made learning and remembering fun."

During this semester I have not only learned new tricks for doing algebra but I have also enjoyed the exchange of ideas with other students. Working in groups has been one of the most enlightening aspects of the course since we have each had the opportunity to become teachers as well as students. Each of us brought a different approach to learning and everyone was willing to share. Since my own personal objective in life is to enter the educational field, I hope to bring many of the ideas I have learned here with me to my own students in the future."

"I would like to say thank you Ted for rekindling my love of math. You always kept your sense of humor, even when I made some foolish mistakes. By erasing my unfounded fears you have given me back my confidence in an area I was sure I was going to fail. By the way I think I did pretty good over-all in this course."

**Student #3**

"I signed up for this class in June with a fairly set idea of what I should expect and how I would use this course to help me in future classes. I had taken algebra before but it was three years ago so I wanted to have a refresher course. I can honestly say that I was not looking forward to the class at all, but the material the teacher sent me prior to the beginning of the semester made me forget about any preconceptions I had about math courses. The course started very smoothly and it was easy to become adjusted to the style of teaching the instructor used."

The class itself was exactly what I was looking for in a math class and I found after a quick review of the material that I remembered it from high school. I found it was more of a math practice rather than a math class, with the class being made up primarily of peer aides and the teacher acting as more of a coach showing the students what they had been doing wrong or how they could do what they were doing right more efficiently."

Conclusion

Cooperative learning techniques, when used extensively in mathematics classes, generate many advantages for the students and teachers. Students' critical thinking skills are enhanced; motivation levels are increased as students become familiar with working with their peers, leading to a new found enjoyment of mathematics classes; achievement levels increase and thus math anxiety is reduced and student self esteem is increased; professors and students get to know each other better as individuals, increasing motivation for both; students form lasting relationships among their peers leading to study groups outside of class and taking follow up classes together. Professors get to learn about their students' backgrounds, abilities and learning styles. Cooperative structures address different student learning styles in every class, including verbal, visual, and kinesthetic. Cooperative learning is especially helpful in encouraging women in the study of mathematics. With all the benefits, it becomes clear why the AMATYC Standards call for cooperative learning paradigms in mathematics classes.

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