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Cooperative Learning Structures Help College Students Reduce Math Anxiety and Succeed In Developmental Courses

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Higher education faces the growing problem of providing remedial mathematics for incoming freshmen, especially at community colleges. Despite efforts by the Federal government and the states to improve mathematics education through the use of standardized competency tests, more incoming students are placed in developmental math courses over time. These courses may include arithmetic, pre-algebra, algebra, and intermediate algebra. Offered as standard high school sequences, placement tests such as Acquplacer put as many as 95% of CCC’s new students into one of these courses. Not only are more students underprepared in math, but many exhibit strong math anxieties because of previous failures. Their anxiety often prevents them from completing their developmental math courses, and thus their college careers are either substantially lengthened or terminated prematurely. Many in the college community view developmental mathematics as a gate keeper course that prevents students from completing their degrees. This paper will describe my approach to helping students overcome their math anxiety through the use of cooperative learning structures, an approach that leads to higher retention rates and success in developmental math courses.

Causes of Math Anxiety

Mathematics anxiety has been defined as feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations. Math anxiety can cause one to forget and lose one’s self-confidence (Tobias, S., 1993). Research confirms that the pressure of timed tests and the risk of public embarrassment cause many students unproductive tension. Three specific practices— common in traditional mathematics classroom—cause enormous anxiety: (1) imposed authority, (2) public exposure, and (3) time deadlines. Cooperative learning structures address each of these problems and much more by encouraging students to become active learners, take more responsibility for their learning, and become more involved in the course procedures. Cooperative learning approaches encourage students not to rely on their instructors for all their learning.

Deborah Russell, (2008, pg1) has captured the essence of the causes of math anxiety. Math anxiety or fear of math is actually quite common. Math anxiety is quite similar to stagefright. Why does someone suffer stagefright? Fear of something going wrong in front of a crowd? Fear of forgetting the lines? Fear of being judged poorly? Fear of going completely blank? Math anxiety conjures up fear of some type. The fear that one won't be able to do the math or the fear that it's too hard or the fear of failure which often stems from having a lack of confidence. For the most part, math anxiety is the fear about doing the math right, our minds draw a blank and we think we'll fail and of course the more frustrated and anxious our minds become, the greater the chance for drawing blanks. Added pressure of having time limits on math tests and exams also cause the levels of anxiety grow for many students.

Usually math anxiety stems from unpleasant experiences in mathematics. Typically, math phobics have experienced math in crippling ways that limited understanding. Thus, math anxiety often results from poor teaching. Consequently, many students with math anxiety focus on math procedures rather than actually understanding the mathematical concepts. When one tries to memorize procedures, rules and routines without much understanding, the math is quickly forgotten, and panic soon sets in. The division of fractions provides a classic case. Typically, the procedures related to reciprocals and inverses are taught without regard for the underlying concepts: In other words, “It's not yours to reason why, just invert and multiply”. Students who memorize the rule, find that it works. But, do they understand why it works? Did their teachers ever use pizzas or math manipulatives to show them why it works? If not, then, they simply memorized the procedure and that was that. They are fine until they forget a few procedures. At that point, they need to understand the underlying concepts. Once students realize they can do the math because they understand it, they will overcome the math anxiety. Both teachers and parents—any “coaches”—must ensure that students truly understand the math being presented to them. (Russell, 2008, Pg 1)

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Recommendations for Decreasing Student Math Anxiety

Because many students experience math anxiety in the traditional classroom, professors should design them to make students feel more successful. Students must have at best a high level of success or, at worst, a level of failure they can tolerate. Therefore, teachers must handle incorrect responses positively to encourage student participation and enhance their confidence. Studies have shown students learn best when they are active rather than passive learners (Spikell, 1993). Additionally, the theory of multiple intelligences addresses the different learning styles. To reach a majority of students, math lessons should be designed to appeal to different modalities: visual/spatial, logical/mathematics, musical, body/kinesthetic, interpersonal and intrapersonal and verbal/linguistic. Every student is capable of learning, but they may learn in different ways. Therefore, lessons must be presented in a variety of ways. For example, new concepts can introduced by using cooperative groups, visual aids, hands on activities and technology. Learners are different than they were forty years ago when students rarely questioned the “why” of math concepts and focused instead on memorizing and mechanically performing the necessary operations. Today’s learners are often more demanding, wanting to know why something is done this way or that way?

Because students today need “practical” math, it needs to be relevant to their everyday lives. Students enjoy experimenting. To learn mathematics deeply, students must be engaged in exploring, conjecturing, and thinking rather than merely learning by rote the rules and procedures. Cooperative groups give students a chance to exchange ideas, to ask questions freely, to explain concepts to one another, to clarify ideas in meaningful ways and to express feelings about their learning. These critical thinking skills will be beneficial throughout their adult working life.

Cooperative learning Methodology

To establish a positive tone for each upcoming class, two weeks prior to the start of the semester I send my students a letter with a humorous introduction to the class and cooperative learning, a course syllabus, and a writing assignment in the form of a mathematical autobiography. I ask them to read the first chapter and start working on the text problems. The first chapter includes review materials from the prerequisite course. I want to emphasize the students' responsibility in the learning process well before the class starts and to demonstrate my own interest in helping them become independent mathematics learners. I include my home and work phone numbers and email addresses to encourage students to contact me if they have any questions or concerns about working in groups or studying material before class.

**Ed Nufer (this volume, pXXX) stresses the importance of the first class meeting of the semester. “We need a *very* good plan for first day of class because the tenor we set there will be what students will expect to continue all term. Students may not know how to work well together initially to do a learning exercise we design. We have to enlist their interest and guide them through unfamiliar learning exercises. This means we need to start their learning how to learn in the same way that we start their learning a new discipline; start small, start simple, and strive to leave no person behind. This focus must continue throughout the term as we introduce additional cooperative learning exercises, such as the examples that follow.** “

In order to accomplish this I ask the students, seated at tables with four to six to a group, to interview a student sitting next to them who they do not know. I want them to begin the process of interacting with other students whom they are not familiar with using a cooperative structure called Pair Interviews/Introductions. Three specific questions I suggest are: How do they feel about math and why? What concerns do they have about this course? Why are you taking this course at this time? In addition I request they obtain as much biographical information as possible such as majors, employment, family, pets, hobbies, travel, where they live, etc. I want the students to find as many commonalities as possible. Next I ask each pair to introduce their new “math buddy” to their table partners in order to practice what they will say to the whole class. Finally each student introduces his/her partner to the whole class. The students discover a very important affective response. They are not alone in their math anxiety, fear of failure in this course and concern about previous bad math experiences. It actually becomes humorous to hear the repeated statements that students are in the class because they are required to be and not out of choice since this course is a prerequisite for many other science and math courses they need to graduate. There is a palpable sense of relief that I seem to understand their feelings as demonstrated by the nature of the cooperative exercise and my reassuring responses to their stated fears. I use this interview process during the semester to help students ascertain how thoroughly they and their partners understand about concepts we are working on.

On the first day of the semester I distribute a class schedule that specifies which textbook sections they are responsible for on a given day. I encourage students to try to understand the chapter concepts and procedures before coming to class. To accomplish this, I assign worksheets that ask students to write out the chapter section’s outcomes, record crucial vocabulary definitions, and solve 6-10 problems from the section exercises. I count these pre-class worksheets as extra credit in order to reduce the students’ anxiety over being asked to study and apply the material before receiving a lecture from me. I also ask the students to attempt to solve as many problems from the section exercises as possible, after class. Their student manuals and texts provide worked-out solutions for all the odd numbered problems, allowing them to check their work. For the even-numbered problems, I encourage students to check their solutions prior to class by going to the mathematics tutorial center or by consulting with classmates.

During class, students sit at tables with four to six to a group. At the beginning of each class, I give a short explanation of the concept or procedure we will work on. I try to keep my lectures under ten minutes, usually targeting five minutes. I distribute work sheets with problems or questions covering the day's topic or cooperative activity. Students first attempt to complete the worksheet problems in pairs, progressing from simpler problems to more complex ones. Pair work creates an optimal learning environment because one student is explaining while the other is listening. Thus, all the students in the class are participating actively by listening or talking about mathematics. After the pairs complete the assignment, they share their results with the other pairs of students at their table. This paired sharing results provides additional repetition and feedback for the students. As the opening chapter, points out, “Learning is defined as stabilizing, through repeated use, certain appropriate and desirable synapses in the brain,” Leamnson, 2000, p. 5. I place the problems on the board in sequence and ask groups to present their solutions to the class. I also ask students to work directly out of the text together. The text I use has a workbook format that encourages an interactive approach to studying mathematics. When students read the text together and explain sections to their partners, they also have an excellent opportunity to build their mathematics vocabulary.

During each class I circulate around the room observing each group's progress. I make suggestions about how they might go about finding the answers to their questions, but I do not directly answer questions initially. Instead, I encourage the students to use as resources both their texts and any other student or group in the class. Those who did not do the reading and practice problems beforehand have an opportunity to do so at this point in the class. If most students appear to be having difficulty or are making fundamental mistakes, I ask volunteers to put their solutions on the board while explaining and defending their methodology. With this approach, explanations come from the students’ peers, not from an expert speaking “professorese.” After the student explanations, the groups return to work and try to resolve whatever questions remain. If they are still confused, I facilitate a whole class discussion which usually clarifies the source of their confusion.

In addition to the worksheets, I often give group quizzes as a form of review. First each student works the problems individually. Next, they compare answers within their groups and try to reach agreement. To reduce their threat and to reduce anxiety, I count these quizzes as extra credit. The quizzes help me identify which students have reached a competency level. I encourage those who need it to get extra help outside of class, stressing again their responsibility for their learning as part of the class philosophy. I recommend the Mathematics Learning Center or other tutorial agencies on campus. I also suggest that they work with their peers in study groups or make arrangements to see me.

Cooperative learning allows for flexibility in content coverage. On occasion, I will postpone a test when I see that most students have not mastered the material. With all the outside pressures students face today, they may not be ready to demonstrate their knowledge at the time specified on the syllabus. Putting them through a rigorous test at that point would merely raise anxiety and result in multiple failures. My courses, however, are not open ended. We continue covering the syllabus while the students review the material they will be tested on. By negotiating test schedules, the students become more involved in establishing the course procedures and thus empowered to control their learning environment.

Finally, I give an individual in-class test to maintain the accountability of each student. I use a mastery approach where students have an opportunity to correct their mistakes during the exam, before a final grade is calculated. While the students work on the exam, I walk around the room observing their progress. When they have completed their test, I check it immediately and circle any incorrect answers, without, however, indicating what mistake they made. The students then have an opportunity to make corrections. If they get below an 80% after corrections, then they are required to retake a new test outside of class using the same procedure. If they get above an 80%, then I encourage them to continue making corrections until they have completely corrected the test. Their grade is based upon the final corrected test.

Every step of this cooperative learning paradigm is intended to encourage students to take responsibility for their learning. This mindset creates very high expectations for the students and for me as the course facilitator. I need to provide materials to guide the students through the process, and I work with them to develop appropriate group interaction skills. I am intensely involved in each class as I circulate around the room, talking to students individually, or in pairs, or in larger groups. I also carefully balance the classes between group discussions and individual work. Students respond that the classes fly by. Even though they are exhausted at the end of class, they feel good about what they have accomplished. By the end of the semester, the better students have learned how to become independent learners, their math phobia has all but disappeared, and they actually begin to like math. The less motivated students have learned more math than they ever expected to master. In class the students actively work through the content and grow to understand concepts in ways that makes sense to them because they have developed their own solutions to problems.

The procedures described above have evolved over a long period of time through a process of trial and error. I don’t recommend that new teachers initiate an extensive cooperative learning system without first participating in training programs and conferences dealing with cooperative learning techniques. It takes time for them to develop a comfort level and to gain confidence with cooperative processes. To incorporate cooperative learning in math classes, I recommend that teachers initiate one or two new techniques each semester until they have acquired a full repertoire of activities.

Examples of cooperative activities

To give readers an idea of how cooperative learning is implemented, I will describe three activities: Pair-Reading and Math-Olympics may be used to cover any content area; Factoring-Jigsaw was developed for a specific content area. **Shadle (this volume P(XXX) stresses the need for providing well structured cooperative learning activities “For the cooperative learning in my course to be successful, it needs to be well-structured (Millis and Cottell, 1998). Students need to know what to expect during class time; they also need an understanding of the boundaries around the content and skills I expect they will master in my course. Because I am not lecturing, my delivery cannot provide class meetings with an expected passive structure. Instead, I now work intentionally to have the groups and their activities provide the structure students need to have confidence in the course.”**

As one might expect, pairs of students work together on the Pair-Reading exercise. First, both students read the same section of either the text or instructor-provided materials. Next, one student explains a single paragraph or short section of the text to the partner. The partner listens, and then asks questions if he or she does not understand the explanation. The listener then rephrases what he or she heard. The students alternate roles of "explainer" and "listener" until they complete all the material being studied. When the entire class has completed the exercise, I ask groups at random to explain the material to the whole class. These explanations serve as a check to make sure the students do indeed understand the material they are reading. To prepare the students for this activity, I have the students Pair-Read the syllabus during the second class. The syllabus describes the cooperative nature of the class, the mastery approach to testing, grading, attendance policies, and other topics pertinent to the operation of the class. I have found that students initially read through course materials very casually, often missing key elements of course policies. This activity causes students to read through the syllabus carefully and to think critically about each element because they must explain each paragraph to their partners or listen to their explanations. This syllabus activity also encourages students to work with their neighbors, and it begins the process of training students in cooperative learning. Once we have practiced this approach on the syllabus, which is a relatively simple document, I continue to have my students do paired readings in the textbook or using extra handouts that clarify mathematical concepts. **Cohen (this volume) emphasizes the value in working with her students starting with the first class to emphasize that the students are expected to make strong efforts in the cooperative learning activities and in learning in general**. **She uses her syllabus and ice breaking activities during the first week and beyond to accomplish this goal**. Anecdotally, like many other professors, I am observing that more and more students read less, especially when it comes to math textbooks. In fact, as Hobson notes, “A consistent pattern of research findings has established compliance with course reading at 20-30% for any given day and assignment (Burchfield & Sappington, 2000; Hobson, 2003; Marshall, 1974; Self,

1987). The pair reading exercise helps students overcome their disinterest in reading and assures me that they have read the material.

The Math Olympics activity can be used with any content where multiple problems are involved. It is especially useful for chapter reviews or section practice. I use this activity, as an example, to introduce solving equations in elementary algebra classes. The class is divided into groups of four. Existing groups may be used or new groups formed. I place five questions on the board. I use one more question than there are students in the group to discourage having the groups simply divide up the questions, one for each student. After 5-10 minutes, depending on the complexity of the problems, I ask each group to send one student to the board to record their group's answers on a grid I have drawn on the board. I check all the answers. This process can be repeated for the duration of the class or for a portion of it only. If I see that groups are having trouble with a set of problems, I will stop the activity and facilitate a whole class discussion or give a mini-lecture on the material. Students are actively involved in solving many problems in a short period of time during class. I encourage groups to work out their own processes for solving each set of problems. Thus, the students assume some of the responsibility for the class process. I have an opportunity to observe the students solving problems individually and in groups.

**Maier et al (this volume pXXX) identifies research which shows that students working together solve problems more accurately. Research by mathematics educators Vidakovic (1997) and Vidakovic and Martin (2004)) shows that groups are able to solve problems more accurately than individuals working alone. Even when one member is more skilled, the collective group is able to correct errors that would remain unnoticed when the skilled problem solver works independently without explaining his or her procedure.**

The Factoring-Jigsaw activity can be used whenever material can be segmented into seperate components. Each group member becomes an expert on a different concept or procedure and teaches their concept to the group. I use this activity when covering factoring of polynomials, where the coefficient of the first term is one. There are four unique cases. The second and third coefficients may be both positive or both negative or have opposite signs. I form base groups of four students. Students count off from one to four. I distribute a worksheet for each case. The worksheets have five sample polynomials to factor, plus a space for each student to make up five problems of their own. Students reform groups by combining with other students who have been assigned the same case number, again four to a group. The students work together to determine what is unique about their case. They are in effect becoming experts in their case. In the next step the students develop a teaching strategy to bring back to their base groups. In this stage they make up their own problems. Each student practices his/her explanation with the case group. Finally, the students return to their base groups and teach their case. There is no preconceived way in which students must teach their material, so the results are quite varied. This activity helps students understand what teaching mathematics involves as well as providing an interesting and often entertaining class. Students learn how to work with different partners and begin to see that they can indeed assume responsibility for their own learning.

**Maier (this volume p(XXX) identifies a problem learners have trouble retaining a large amounts of information presented over a short time period, as in a lecture and how cooperative learning involves all students continually. “Few learners are able to retain significant portions of a continual steam of new ideas in the absence of breaks or shifts in activity, an observation supported by considerable educational research (Nilson, 2003).” Maier (this volume p(XXX) “Cooperative learning provides an opportunity for *all* students to contribute ideas, critique arguments, and summarize concepts in a relatively short period of time, even in large auditorium-style classrooms. For example, the think-pair-share technique, discussed in other chapters of this volume, can be used in the middle of a lecture to quickly involve all students in a brief problem-solving session or a discussion of a course concept. Because this technique requires equal participation by all students, it is more likely to actively engage the entire class in the learning process.”**

Student responses to my cooperative learning approach

At the end of each semester, I give a writing assignment to get the students to reflect on their performance and behavior in the class. The assignment is also designed to have them think about how they will approach their next math class and to think about needed changes to ensure their success in future classes. Some of the questions I ask are: Has your approach to math changed during this course or compared to previous courses? If yes, How? Have your attitudes or feelings about math changed? How do you feel you performed in this course? What would you do differently in this course if you had it to do all over again? What would you suggest I do differently in my future classes? What else would you like to add that I did not ask?

The responses are very candid, but not always positive, a result I attribute to using cooperative learning techniques throughout the semester that allow me to get to know students and allow them to get to know and trust me. I emphasize that I am never offended by what students write if they are being honest and constructive. Developing this relationship with my students is extremely important to me. Most of the students’ comments about the class and my techniques are positive. I regard this as a testimonial to the nature of cooperative learning which allows me to show the students my human side at the same time that I get to know them as people, not just ciphers in my grade book.

Students wrote about their experiences as follows:

In the past as you know, Ted, I have taken a class with you and have enjoyed your approach in learning the material. Before your classes I disliked math. I was always getting aggravated and scared by it. Working together with those around me in a group has been a great help in understanding the material and the many different ways in which a problem can be tackled and solved. For me the beauty was being able to work one on one with someone every day. I was constantly learning something new and leaving class feeling relaxed and in control. On those days I could not understand something, I did not feel half as bad as I normally would have, I knew that if it were something I could not figure out at home or at the next class period I could count on receiving help.

When I re-entered school almost two years ago I was told that I needed to take an algebra course. I panicked. Even though I had taken a large number of math courses in high school I feared that I had forgotten everything I had learned so long ago. It was a pleasure to realize how much I truly enjoy working with numbers once again. The course was presented in a way that made learning and remembering fun. During this semester I have not only learned new tricks for doing algebra but I have also enjoyed the exchange of ideas with other students. Working in groups has been one of the most enlightening aspects of the course since we have each had the opportunity to become teachers as well as students. Each of us brought a different approach to learning and everyone was willing to share. Since my own personal objective in life is to enter the educational field, I hope to bring many of the ideas I have learned here with me to my own students in the future.

I would like to say thank you Ted for rekindling my love of math. You always kept your sense of humor, even when I made some foolish mistakes. By erasing my unfounded fears you have given me back my confidence in an area I was sure I was going to fail. By the way I think I did pretty good over-all in this course.

Before taking this class I had negative attitudes towards math. I did not understand too much and focused little on learning it. However, my thinking has begun to change. I am able to figure out problems that I once thought were too complicated to complete. You made the atmosphere one in which it was fun to learn. I feel that my performance has improved as a result of this. Students were able to communicate and work through problems together. The grades I received were higher than any other math course I have been in. I was happily surprised.

Overall I would have to say that the laid back, conversational and non-threatening way the course was structured seemed to help me overcome some of my preconceived notions about math. The course was set up in such a way that made learning a little more fun than in previous math classes. Also, being able to converse openly to my neighbor, or the teacher, if I had a problem, certainly helped me feel relaxed, non-threatened, and at ease if I ever had trouble finding an answer to a problem. I had a feeling that if I ever had a problem I could go to the teacher or a student and I could troubleshoot or dig until I got the information I needed. The uninhibited atmosphere in the classroom made me feel that no matter how difficult a particular problem was to grasp, I could use the resources available to me in the classroom without feeling intimidated. Being less intimidated meant that I could ask more and more questions until I got the information that I needed to succeed.

What can you do to make this course better? Well, I am grateful to you for developing your techniques. I was able to grow because of it. But, I feel that a more regular pattern of homework expectancy from you would allow DE students to further cement the practices needed for future college level courses. I realize it is not your job to baby-sit, but the DE students are here to re-learn (or learn for the first time) proper math habits.

Conclusion

Cooperative learning techniques, when used extensively in mathematics classes, generate many advantages for the students and teachers. Students' critical thinking skills are enhanced; motivation levels are increased as students become familiar with working with their peers, leading to a new-found enjoyment of mathematics classes; achievement levels increase and thus math anxiety is reduced and student self esteem is increased; professors and students get to know each other better as individuals, increasing motivation for both; students form lasting relationships among their peers, leading to study groups outside of class and taking follow up classes together. Professors get to learn about their students' backgrounds, abilities and learning styles. Cooperative structures address different student learning styles in every class, including verbal, visual, and kinesthetic. All these benefits improve student math competence and reduce math anxieties, leading to better retention and success in developmental math classes.

**Nufer (this volume pXXX) recommendation regarding starting to use cooperative learning approaches for faculty new to this paradigm is “To learn in-depth conceptual designs of cooperative learning exercises from specialists, the reader should obtain Johnson, Johnson and Smith (2006), Millis and Cottell, (1998) or Cooper, Robinson, and Ball, (2003) and do a thorough cover-to-cover read before picking any exercise crafted by another party.”**

**Smith et al (this volume) provides an extensive bibliography of cooperative learning articles and books.**

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